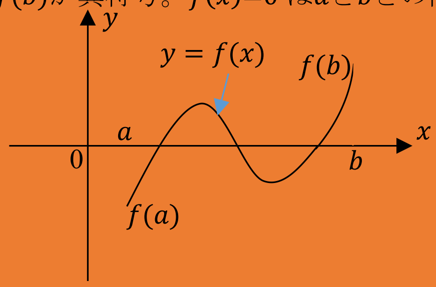
数学Ⅲ　微分・積分　解法のテクニック

1.高次方程式が実数解をもつことの証明。

（解法のテクニック）

高次方程式が実数解をもつことの証明。

➡ $f(a)$と$f(b)$が異符号。$f(x)=0$ はaとbとの間に少なくとも１つの解をもつ



2.数列の極限

（解法のテクニック）

数列の極限

➡ ①分数の形：分母子を分母の最高次で割る。

②無理数の形：有理化を考える。→平方の差を利用。

3. p,qどちらか一方だけが成立することの証明。

（解法のテクニック）

p,qどちらか一方だけが成立することの証明。

➡ p,qどちらも成立すると仮定して矛盾を導く。

かつ，p,qどちらも成立しないと仮定して矛盾を導く。

4. 三角関数の近似➡ラジアンにする。

5. 既約分数の分母が２，５以外の素因数をもつ。→循環小数

(例) $\frac{1}{3},\frac{1}{7}$

6. $\lim_{n\to\infty}\sum_{k=1}^{n} g(n,k)$ の求め方

$\rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}f\left(\frac{k}{n}\right)=\int_0^1 f(x)dx$ を利用する。

7.シュワルツの不等式$(a<b)$

$$\left\{\int_a^b f(x)g(x)dx\right\}^2 \leq \left(\int_a^b \{f(x)\}^2 dx\right)\left(\int_a^b \{g(x)\}^2 dx\right)$$

[証明]

任意の実数tにおいて

$\{tf(x)+g(x)\}^2 \geq$ …①

$a<b$より

$$\int_a^b \{\{tf(x)+g(x)\}\}^2 dx \geq 0$$

$\int_a^b \{f(x)\}^2 dx\, t^2 + 2\int_a^b f(x)g(x)dx\, t + \int_a^b \{g(x)\}^2 dx \geq 0$

(ⅰ) $\int_a^b \{f(x)\}^2 dx > 0 \Leftrightarrow f(x) \neq 0$ のとき

判別式$\frac{D}{4}=\left\{\int_a^b f(x)g(x)dx\right\}^2-\left(\int_a^b \{f(x)\}^2 dx\right)\left(\int_a^b \{g(x)\}^2 dx\right)\leq 0$

$$\therefore \left\{\int_a^b f(x)g(x)dx\right\}^2 \leq \left(\int_a^b \{f(x)\}^2 dx\right)\left(\int_a^b \{g(x)\}^2 dx\right)$$

等号成立は$g(x)=0$ のとき

または$\frac{D}{4}=0$ のとき、すなわち

tについての 2 次方程式$\int_a^b \{f(x)\}^2 dx\, t^2 + 2\int_a^b f(x)g(x)dx\, t + \int_a^b \{g(x)\}^2 dx$…②

が重解を持つとき$\frac{D}{4}=0$ となり等号成立。

(ⅱ) $f(x)=0$ のとき

$$\int_a^b f(x)g(x)dx=0, \int_a^b \{f(x)\}^2 dx = 0$$

となり等号成立。

ゆえに

$$\left\{\int_a^b f(x)g(x)dx\right\}^2 \leq \left(\int_a^b \{f(x)\}^2 dx\right)\left(\int_a^b \{g(x)\}^2 dx\right)$$

等号成立は

$f(x)=0$,または $g(x)=0$ またはtについての 2 次方程式②が重解を持つとき。

7.1 $y=f(x)$ が $[0,1]$ で常に正であるときシュワルツの不等式を用いて

$\int_0^1 f(x)dx\cdot\int_0^1 \frac{1}{f(x)}dx \geq 1$ を証明せよ。

$$\int_0^1 \left\{\sqrt{f(x)}\right\}^2 dx\cdot\int_0^1 \left\{\frac{1}{\sqrt{f(x)}}\right\}^2 dx \geq \int_0^1 \left\{\sqrt{f(x)}\cdot\frac{1}{\sqrt{f(x)}}\right\}^2 dx = 1$$

8.ロピタルの定理

ロピタルの定理

$\lim_{x\to a}\frac{f'(x)}{g'(x)}=l$（有限確定値）ならば $\lim_{x\to a}\frac{f(x)}{g(x)}=l$

［証明］

［コーシーの平均値の定理］

$f(x),g(x)$ が $a\leq x\leq b$ で連続かつ $g(a)\neq g(b)$ $a\leq x\leq b$ で微分可能かつ $g'(x)\neq 0$

ならば

$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$ となる $a<c<b$ であるような実数 c が存在する。

（コーシーの平均値の定理の証明）

$F(x)=f(x)-f(a)-\frac{f(b)-f(a)}{g(b)-g(a)}(g(x)-g(a))$

$F(b)=F(a)$

より，ロルの定理より $F'(c)=0(a<c<b)$ が得られる。

コーシーの平均値の定理より

$f(a)=g(a)=0$

$\frac{f(x)}{g(x)}=\frac{f(x)-f(a)}{g(x)-g(a)}=\frac{f'(c)}{g'(c)}, a<c<x$ または $x<c<a$

$x\to a$ のとき $c\to a$ となるから

$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$

例 1.

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}=1$$

ロピタルの定理を用いると

$$\lim_{x\to 0}\frac{(\tan x)'}{x'}=\lim_{x\to 0}\frac{1}{\cos^2 x}=1$$

例 2.

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim_{x\to a}(\sin c)'(a<c<x)=(\sin a)'=\cos a$$

ロピタルの定理を用いると

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim_{x\to a}\frac{\cos x}{1}=\cos a$$

関数の微分公式

1. $\left(x^n\right)'=nx^{n-1}$

2. $(\sin x)'=\cos x$

3. $(\cos x)'=-\sin x$

4. $(\tan x)'=\dfrac{1}{\cos^2 x}$

［証明］

$$\left(\tan x\right)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{\cos x\cdot\cos x-\sin x(-\sin x)}{\cos^2 x}=\frac{1}{\cos^2 x}$$

5. $\left(e^x\right)'=e^x$

6. $\left(e^{-x}\right)'=-e^{-x}$

7. $\left(a^x\right)'=(\log a)a^x$

8. $\left(\log x\right)'=\dfrac{1}{x}$

9. $\{\log f(x)\}' = \dfrac{f'(x)}{f(x)}$

10. $(uv)' = u'v + uv'$

11. $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

関数の不定積分　**(積分定数は省略)**

1. $\int x^n dx = \dfrac{1}{n+1} x^{n+1}$

2. $\int \dfrac{1}{x} dx = \log |x|$

3. $\int \sin x dx = -\cos x$

4. $\int \cos x dx = \sin x$

5. $\int \dfrac{f'(x)}{f(x)} dx = \log|f(x)|$

6. $\int \log x dx = x \log x - x$

7. $\int e^x dx = e^x$

8. $\int a^x dx = \dfrac{a^x}{\log a}$

積分公式

【三角関数】

$\bullet \int \sin^n x dx = \int \sin^{n-1} x \sin x dx$

$\cos x = t, -\sin x dx = dt$

$\bullet \int \cos^n x dx = \int \cos^{n-1} \cos x dx$

$\sin x = t, \cos x dx = dt$

$\bullet \tan\frac{x}{2} = t, \frac{\frac{1}{2}dx}{\cos^2\frac{x}{2}} = dt \Rightarrow dx = \frac{2}{1+t^2}dt, \tan x = \frac{2t}{1-t^2}, \cos x = 2\cos^2\frac{x}{2} - 1 = \frac{2}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$

$\sin x = \cos x \tan x = \frac{2t}{1+t^2}$

例 1. $\int \frac{dx}{1-\cos x}$

$$\tan\frac{x}{2} = t \text{ とおくと } \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2}{1+t^2}dt$$

$$\int \frac{dx}{1-\cos x} = \int \frac{1}{1-\left(\frac{1-t^2}{1+t^2}\right)}\frac{2}{1+t^2}dt$$

$$= \int \frac{1}{t^2}dt = -\frac{1}{t} = -\frac{1}{\tan(x/2)}$$

例 2. $\int \frac{dx}{\cos x}$

$$\int \frac{dx}{\cos x} = \int \frac{\cos x}{\cos^2 x}dx$$

$$= \int \frac{\cos x}{1-\sin^2 x}dx$$

$$\sin x = t \text{とおく。} \cos x\, dx = dt$$

$$\int \frac{dt}{1-t^2} = -\int \frac{dt}{t^2-1} = -\int \frac{1}{2}\left\{\frac{1}{t-1} - \frac{1}{t+1}\right\}dt = -\frac{1}{2}\{\log(t-1) - \log(t+1)\}$$

$$= \frac{1}{2}\log\left|\frac{\sin x + 1}{\sin x - 1}\right|$$

例 3. $\int \cos^2 x dx = \int \frac{\cos 2x + 1}{2} dx = \frac{\sin 2x}{4} + \frac{x}{2}$

例 4.

$$\int \cos^3 x dx = \int (1 - \sin^2 x) \cos x dx$$
$$\sin x = t, \cos x dx = dt$$
$$= \int (1 - t^2) dt = t - \frac{t^3}{3} = \sin x - \frac{\sin^3 x}{3}$$

例 5. $\int \cos^3 x \sin x dx$

$\cos x = t$ とおく。$-\sin x dx = dt$

$$\int \cos^3 x \sin x dx = \int t^3 (-dt) = -\frac{t^4}{4} = -\frac{\cos^4 x}{4}$$

例 6.

$$\int \frac{\cos^2 x}{1 + \sin x} dx = \int \frac{1 - \sin^2 x}{1 + \sin x} dx = \int 1 - \sin x dx = x + \cos x$$

例 7.

$$\int \tan^4 x dx = \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx = \int \tan^2 x \frac{1}{\cos^2 x} dx - \int \tan^2 x dx$$
$$= \frac{1}{3} \tan^3 x - \int \frac{1}{\cos^2 x} - 1 dx = \frac{1}{3} \tan^3 x - \tan x + x$$

例 8.

$$\int \frac{1}{1 + \tan x} dx = \int \frac{1}{1 + \frac{\sin x}{\cos x}} dx = \int \frac{\cos x}{\cos x + \sin x} dx$$
$$= \int \frac{\cos x \cos x - \cos x \sin x}{(\cos x + \sin x)(\cos x - \sin x)} dx = \int \frac{\frac{\cos 2x + 1}{2} - \frac{\sin 2x}{2}}{\cos 2x} dx = \frac{1}{2} \int 1 + \frac{1}{\cos 2x} + \frac{-\sin 2x}{\cos 2x} dx$$
$$= \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{2} \int \frac{1}{\cos 2x} dx = \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{2} \int \frac{\cos 2x}{\cos^2 2x} dx$$
$$= \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{2} \int \frac{\cos 2x}{1 - \sin^2 2x} dx = \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{2} \int \frac{1}{2} \cdot \frac{dt}{1 - t^2}$$
$$= \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{4} \int \frac{1}{2} \left(\frac{1}{1 - t} + \frac{1}{1 + t} \right) dt = \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{8} \int -\frac{1}{t - 1} + \frac{1}{t + 1} dt$$
$$= \frac{x}{2} + \frac{1}{4} \log|\cos 2x| + \frac{1}{8} \log \left| \frac{\sin 2x + 1}{\sin 2x - 1} \right|$$

例 9.

$$\int_0^3 \sqrt{9-x^2}dx, x = 3\sin\theta, dx = 3\cos\theta d\theta$$

$$= 3\int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}3\cos\theta d\theta = 9\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta = 9\int_0^{\frac{\pi}{2}} \frac{\cos 2\theta + 1}{2} d\theta$$

$$= \frac{9}{2}\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\frac{\pi}{2}} = \frac{9}{4}\pi$$

例 10.

$$\int \frac{e^{2x}}{e^x - 1} dx, e^x = t, e^x dx = dt$$

$$= \int \frac{t}{t-1} dt = \int \frac{t-1+1}{t-1} dt = t + \log|t-1| = e^x + \log|e^x - 1|$$

例 11.

$$\int \frac{\log x}{x(\log x + 1)} dx, \log x = t, \frac{dx}{x} = dt$$

$$= \int \frac{t}{t+1} dt = \int \frac{t+1-1}{t+1} dt = t - \log|t+1| = \log x - \log|\log x + 1|$$

例 12.

$$\int \frac{1}{3\sin x + 4\cos x} dx = \int \frac{1}{5\sin(x+\alpha)} dx = \frac{1}{5}\int \frac{\sin(x+\alpha)}{\sin^2(x+\alpha)} dx$$

$$= \frac{1}{5}\int \frac{\sin(x+\alpha)}{1-\cos^2(x+\alpha)} dx = \frac{1}{5}\int \frac{-dt}{1-t^2} = \frac{1}{5}\int -\frac{1}{2}\left(\frac{1}{1-t} + \frac{1}{1+t}\right) dt$$

$$= \frac{1}{10}\int \frac{1}{t-1} - \frac{1}{t+1} dt = \frac{1}{10}\log\left|\frac{\sin(x+\alpha)-1}{\sin(x+\alpha)+1}\right|$$

例 13.

$$\int \tan x dx = -\int \frac{-\sin x}{\cos x} dx = -\log|\cos x|$$

例 14.

$$\int \frac{1}{1+\sin x} dx = \int \frac{1-\sin x}{1-\sin^2 x} dx = \int \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} dx = \tan x - \frac{1}{\cos x}$$

例 15.

$$\int \frac{\sin 2x}{\sqrt{1+\sin^2 x}}dx = \int \frac{2\sin x \cos x}{\sqrt{1+\sin^2 x}}dx = 2\sqrt{1+\sin^2 x}$$

例 16.

$$\int \frac{1}{2+\sqrt{3}\cos x+\sin x}dx = \int \frac{1}{2+2\sin\left(x+\frac{\pi}{3}\right)}dx = \frac{1}{2}\int \frac{1-\sin\left(x+\frac{\pi}{3}\right)}{1-\sin^2\left(x+\frac{\pi}{3}\right)}dx$$

$$= \frac{1}{2}\int \frac{1-\sin\left(x+\frac{\pi}{3}\right)}{\cos^2\left(x+\frac{\pi}{3}\right)} = \frac{1}{2}\int \frac{1}{\cos^2\left(x+\frac{\pi}{3}\right)} + \frac{-\sin\left(x+\frac{\pi}{3}\right)}{\cos^2\left(x+\frac{\pi}{3}\right)}dx = \frac{1}{2}\tan\left(x+\frac{\pi}{3}\right) - \frac{1}{2}\cdot\frac{1}{\cos\left(x+\frac{\pi}{3}\right)}$$

【オイラーの置換】

$\sqrt{x^2+A} \Rightarrow x+\sqrt{x^2+A}=t$ とおく。

例 17. $\int \frac{dx}{\sqrt{x^2+2}}$

オイラーの置換　$\sqrt{x^2+2}+x=t$とおく。

$$x^2+2 = x^2-2tx+t^2$$

$$x = \frac{t^2-2}{2t} = \frac{t}{2}-\frac{1}{t}$$

$$\frac{dx}{dt} = \frac{1}{2}+\frac{1}{t^2}$$

$$(与式) = \int \frac{1}{t-\left(\frac{t}{2}-\frac{1}{t}\right)}\left(\frac{1}{2}+\frac{1}{t^2}\right)dt = \int \frac{dt}{t} = \log|t| = \log\left(x+\sqrt{x^2+2}\right)$$

例 18.

$$\int \frac{\cos x}{\sqrt{1+\sin^2 x}}dx$$

$\sin x = t$ とおく。$\cos x dx = dt$

$$\int \frac{dt}{\sqrt{1+t^2}} = \log(t+\sqrt{t^2+1}) = \log(\sin x + \sqrt{\sin^2 x + 1})$$

【部分積分】

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

例 19. $\int e^x \sin x dx$

$$I = \int e^x \sin x dx = \int e^x (-\cos x)' dx = -e^x \cos x - \int e^x (-\cos x) dx$$
$$= -e^x \cos x + \int e^x (\sin x)' dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$
$$= -e^x \cos x + e^x \sin x - I$$
$$I = \frac{1}{2}\left(-e^x \cos x + e^x \sin x\right)$$

例 20.

$$\int x^2 \log x dx = \int \left(\frac{x^3}{3}\right)' \log x dx = \frac{x^3}{3}\log x - \int \frac{x^3}{3}\cdot\frac{1}{x}dx$$
$$= \frac{x^3}{3}\log x - \frac{1}{3}\cdot\frac{x^3}{3} = \frac{x^3}{3}\log x - \frac{x^3}{9}$$

例 21.

$$\int \log(x^2+1)dx = \int x' \log(x^2+1)dx$$
$$= x\log(x^2+1) - \int x\frac{2x}{x^2+1}dx$$
$$= x\log(x^2+1) - 2\int \frac{x^2+1-1}{x^2+1}dx = x\log(x^2+1) + 2\int -1 + \frac{1}{x^2+1}dx$$
$$= x\log(x^2+1) - 2x + 2\int \frac{1}{x^2+1}dx = x\log(x^2+1) - 2x + 2\int \frac{1}{\tan^2\theta+1}\cdot\frac{d\theta}{\cos^2\theta}$$
$$= x\log(x^2+1) - 2x + 2\theta$$

例 22.

$$\int x\log(x+1)dx$$

$$X=x+1$$

$$\int(X-1)\log XdX=\int(X\log X-\log X)dX=\int X\log XdX-(X\log X-X)$$

$$=\int\left(\frac{X^2}{2}\right)'\log XdX-X\log X+X=\frac{X^2}{2}\log X-\int\frac{X^2}{2}\cdot\frac{1}{X}dX-X\log X+X$$

$$=\frac{X^2}{2}\log X-\frac{X^2}{4}-X\log X+X=\frac{(x+1)^2}{2}\log(x+1)-\frac{(x+1)^2}{4}-(x+1)\log(x+1)+x+1$$

例 23.

$$\int x\log\left(x^2+x+\frac{1}{4}\right)dx=\int x\log\left(x+\frac{1}{2}\right)^2dx$$

$$x+\frac{1}{2}=X$$

$$\int\left(X-\frac{1}{2}\right)\log X^2dX=\int X\log X^2-\frac{1}{2}\log X^2dX$$

$$=\int\left(\frac{X^2}{2}\right)'\log X^2dX-\frac{1}{2}\int X'\log X^2dX$$

$$=\frac{X^2}{2}\log X^2-\int\frac{X^2}{2}\cdot\frac{2X}{X^2}dX-\frac{1}{2}X\log X^2+\frac{1}{2}\int X\cdot\frac{2X}{X^2}dX$$

$$=\frac{X^2}{2}\log X^2-\frac{X^2}{2}-\frac{1}{2}X\log X+X$$

$$=\frac{(x+1)^2}{2}\log(x+1)^2-\frac{(x+1)^2}{2}-\frac{1}{2}(x+1)\log(x+1)+x+1$$

例 24.

$$\int \frac{x}{\cos^2 x}dx = \int x(\tan x)' dx = x\tan x - \int \tan x dx$$

$$= x\tan x + \int \frac{-\sin x}{\cos x}dx = x\tan x + \log|\cos x|$$

例 25.

$$\frac{\pi}{2\sqrt{2}}\int_0^2\left(t^2-2t\log(t+1)+\{\log(t+1)\}^2\right)\left(\frac{t+2}{t+1}\right)dt$$

$$=\frac{\pi}{2\sqrt{2}}\int_0^2\left(\frac{t^3+2t^2}{t+1}-2\left(t^2+2t\right)\log(t+1)\frac{1}{t+1}+(t+2)\{\log(t+1)\}^2\frac{1}{t+1}\right)dt$$

$$=\frac{\pi}{2\sqrt{2}}\int_0^2\frac{t^3+t^2+t^2+t-t-1+1}{t+1}dt-\frac{\pi}{\sqrt{2}}\int_0^2\frac{t^2+2t}{t+1}\log(t+1)dt$$

$$+\frac{\pi}{2\sqrt{2}}\int_0^2\{t+\log(t+1)\}'\{\log(t+1)\}^2dt$$

$$=\frac{\pi}{2\sqrt{2}}\int_0^2 t^2+t-1+\frac{1}{t+1}dt-\frac{\pi}{\sqrt{2}}\int_0^2\frac{t^2+t+t+1-1}{t+1}\log(t+1)dt$$

$$+\frac{\pi}{2\sqrt{2}}\left[\{t+\log(t+1)\}\{\log(t+1)\}^2\right]_0^2-\frac{\pi}{\sqrt{2}}\int_0^2\{t+\log(t+1)\}\log(t+1)\frac{1}{t+1}$$

$$=\frac{\pi}{2\sqrt{2}}\left[\frac{t^3}{3}+\frac{t^2}{2}-t+\log(t+1)\right]_0^2-\frac{\pi}{\sqrt{2}}\int_0^2\left(t+1-\frac{1}{t+1}\right)\log(t+1)dt$$

$$+\frac{\pi}{2\sqrt{2}}\left[2(\log 3)^2+(\log 3)^3\right]-\frac{\pi}{\sqrt{2}}\int_0^2\left\{\left(1-\frac{1}{t+1}\right)\log(t+1)+\{\log(t+1)\}^2\frac{1}{t+1}\right\}dt$$

$$=\frac{\pi}{2\sqrt{2}}\left[\frac{8}{3}+\log 3\right]-\frac{\pi}{\sqrt{2}}\int_0^2\left(\frac{t^2}{2}+t\right)'\log(t+1)+\frac{\pi}{\sqrt{2}}\int_0^2\log(t+1)\frac{1}{t+1}dt$$

$$+\frac{\pi}{2\sqrt{2}}\left[2(\log 3)^2+(\log 3)^3\right]-\frac{\pi}{\sqrt{2}}\left[(t+1)\log(t+1)-t\right]_0^2+\frac{\pi}{\sqrt{2}}\left[\frac{1}{2}\{\log(t+1)\}^2-\frac{1}{3}\{\log(t+1)\}^3\right]_0^2$$

$$=\frac{\pi}{2\sqrt{2}}\left[\frac{8}{3}+\log 3\right]-\frac{\pi}{\sqrt{2}}\left[\left(\frac{t^2}{2}+t\right)\log(t+1)\right]_0^2+\frac{\pi}{\sqrt{2}}\int_0^2\left(\frac{t^2}{2}+t\right)\frac{1}{t+1}dt+\frac{\pi}{\sqrt{2}}\left[\frac{1}{2}\{\log(t+1)\}^2\right]_0^2$$

$$+\frac{\pi}{2\sqrt{2}}\left[2(\log 3)^2+(\log 3)^3\right]-\frac{\pi}{\sqrt{2}}3\log 3+\frac{2\pi}{\sqrt{2}}+\frac{\pi}{\sqrt{2}}\left[\frac{1}{2}(\log 3)^2-\frac{1}{3}(\log 3)^3\right]$$

$$=\frac{\pi}{2\sqrt{2}}\left[\frac{8}{3}+\log 3\right]-\frac{\pi}{\sqrt{2}}\left[4\log 3\right]+\frac{\pi}{2\sqrt{2}}\int_0^2\frac{t^2+2t}{t+1}dt+\frac{\pi}{\sqrt{2}}\left[\frac{1}{2}(\log 3)^2\right]$$

$$+\frac{\pi}{2\sqrt{2}}\left[2(\log 3)^2+(\log 3)^3\right]-\frac{\pi}{\sqrt{2}}3\log 3+\frac{2\pi}{\sqrt{2}}+\frac{\pi}{\sqrt{2}}\left[\frac{1}{2}(\log 3)^2-\frac{1}{3}(\log 3)^3\right]$$

$$=\frac{10\pi}{3\sqrt{2}}-\frac{13\pi}{2\sqrt{2}}\log 3+\frac{2\pi}{\sqrt{2}}(\log 3)^2+\frac{\pi}{6\sqrt{2}}(\log 3)^3+\frac{\pi}{2\sqrt{2}}\int_0^2\frac{t^2+t+t+1-1}{t+1}dt$$

$$=\frac{10\pi}{3\sqrt{2}}-\frac{13\pi}{2\sqrt{2}}\log 3+\frac{2\pi}{\sqrt{2}}(\log 3)^2+\frac{\pi}{6\sqrt{2}}(\log 3)^3+\frac{\pi}{2\sqrt{2}}\left[\frac{t^2}{2}+t-\log(t+1)\right]_0^2$$

$$=\frac{10\pi}{3\sqrt{2}}-\frac{13\pi}{2\sqrt{2}}\log 3+\frac{2\pi}{\sqrt{2}}(\log 3)^2+\frac{\pi}{6\sqrt{2}}(\log 3)^3+\frac{\pi}{2\sqrt{2}}\left[4-\log 3\right]$$

$$=\frac{16\pi}{3\sqrt{2}}-\frac{7\pi}{\sqrt{2}}\log 3+\frac{2\pi}{\sqrt{2}}(\log 3)^2+\frac{\pi}{6\sqrt{2}}(\log 3)^3$$