(問題２１６)

角α,β,γが$\alpha+\beta+\gamma=\pi,\alpha\geq 0,\beta\geq 0,\gamma\geq 0$を満たすとき，

$\cos\alpha+\cos\beta+\cos\gamma\geq 1$を示せ。

(解答)

$$
\begin{aligned}
&\cos\alpha+\cos\beta+\cos\gamma-1\\
&=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+\cos(\pi-(\alpha+\beta))-1\\
&=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-\cos(\alpha+\beta)-1\\
&=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-\left(2\cos^2\frac{\alpha+\beta}{2}-1\right)-1\\
&=2\cos\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}\right)\\
&=2\cos\frac{\pi-\gamma}{2}\cdot 2\sin\frac{\alpha}{2}\sin\left(-\frac{\beta}{2}\right)\\
&=4\left(-\sin\frac{\gamma}{2}\right)\sin\frac{\alpha}{2}\sin\left(-\frac{\beta}{2}\right)\\
&=4\sin\frac{\gamma}{2}\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\geq 0\left(\alpha+\beta+\gamma=\pi,\alpha\geq 0,\beta\geq 0,\gamma\geq 0\Rightarrow 0\leq\alpha,\beta,\gamma\leq\pi\right)
\end{aligned}
$$